today:

turn in graphs for webwork extra credit project 1
homework 4 due (6.4.10, 6.4.16, 7.1.28, 7.1.56, 7.2.44, 7.2.66)
quiz: §§ 6.4, 7.1
§ 7.3 - trig substitution
review

wednesday:

webwork 4 due @ 11:55 pm
mslc: webwork workshop @ 12:30, 1:30, 2:30, 3:30, 4:30 in SEL 040
mslc: integration techniques workshop @ 1:30 and 3:30 in CH 042
mslc: midterm review @ 7:30 pm in HI 131

thursday:

midterm: §§ 6.2-6.4, 7.1-7.3
§ 7.4 - partial fractions

tuesday, 3 november:

homework 5 due (7.3.8, 7.3.22, 7.3.40, 7.4.20, 7.4.48, 7.4.50)
§ 4.4 - l'Hôpital's rule

thursday, 5 november:

§ 7.8 - improper integrals

friday, 6 november:

webwork 5 due @ 11:55 pm
last drop day

review

last time we learned how to integrate many trig functions.

For example, we integrate cos^2 x by using the identity cos^2(x)=(1+cos(2x))/2,

then we make an easy u-substitution to finish the problem.

Answer:

x/2 + sin(2x)/4 + C

evaluate

$$\int \cos^2 x \, \mathrm{d}x$$

review

This is another u-substitution problem.

evaluate by making the u-substitution u=1-x^2

evaluate

$$\int 2\,x\,\sqrt{1-x^2}\,\mathrm{d}x$$

why does u-substitution work?

why does u-substitution work?

It works because of the chain rule. Remember:

$$f'\left(g\left(x\right)\right)\,g'\left(x\right)\,\mathrm{d}x = f'(u)\,\mathrm{d}u$$

where $u = g\left(x\right)$.

The chain rule is an equality. For u-substitution, we go left to right, but we could just as easily go right to left if we had some functions g that had identities to help us simplify... for example, trig functions.

example

What is the area of a circle with radius r?

Remember, a circle with radius r centered at the point $(x_0,\, y_0)$ has equation

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = r^2$$

without loss of generality, we suppose the circle is centered at the origin. Then

$$x^2 + y^2 = r^2$$

so we are considering the region bounded by

$$y = \sqrt{r^2 - x^2}$$

and

$$y = -\sqrt{r^2 - x^2}$$

we've dealt with this integral a couple times before by calling it the area of a circle and saying without proof that it was therefore pi r^2. Now we prove it.

thus the area of a circle with radius r is

$$\int_{-r}^{r} 2\sqrt{r^2 - x^2}\,\mathrm{d}x = \int_{-r}^{r} 2\sqrt{r^2\left(1 - (x/r)^2\right)}\,\mathrm{d}x$$

Inside the integral, we have a 1 - something squared. That reminds us of the rule

1 - sin^2 y = cos^2 y

so we substitute

x/r = sin θ

We substitute $x/r = \sin\,\theta$.
Thus $x = r\,\sin\,\theta$, so $\mathrm{d}x = r\,\cos\,\theta\;\mathrm{d}\theta$.

$$
\begin{aligned}
2\int\sqrt{r^2\left(1-(x/r)^2\right)}\,\mathrm{d}x &= 2\int r\,\cos\theta\,\sqrt{r^2\left(1-\sin^2\theta\right)}\,\mathrm{d}\theta \\
&= 2\int r\,\cos\theta\,\sqrt{r^2\cos^2\theta}\,\mathrm{d}\theta \\
&= 2\int r^2\,\cos^2\theta\,\mathrm{d}\theta \\
&= 2\,r^2\int\cos^2\theta\,\mathrm{d}\theta \\
&= r^2\left(\theta+\frac{1}{2}\sin\left(2\theta\right)\right)+C
\end{aligned}
$$

In u-substitution, we set
u=f(x).

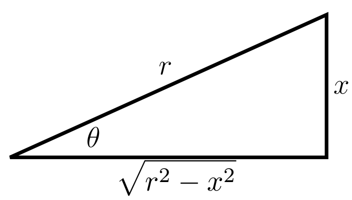
Here we set
x=f(θ)

(This is the reverse.)

The integral of
cos^2 was our first
example.

We're not done, since the rhs is in
terms of two variables.
How do we change back to r?

We substitute $x/r = \sin\,\theta$.
Thus $x = r\,\sin\,\theta$, so $\mathrm{d}x = r\,\cos\,\theta\;\mathrm{d}\theta$.



$$
\begin{aligned}
2\int\sqrt{r^2\left(1-(x/r)^2\right)}\,\mathrm{d}x &= r^2\left(\theta+\frac{1}{2}\sin\left(2\theta\right)\right)+C \\
&= r^2\left(\theta+\sin\left(\theta\right)\cos\left(\theta\right)\right)+C \\
&= r^2\left(\sin^{-1}\left(\frac{x}{r}\right)+\frac{x}{r}\,\frac{\sqrt{r^2-x^2}}{r}\right)+C
\end{aligned}
$$

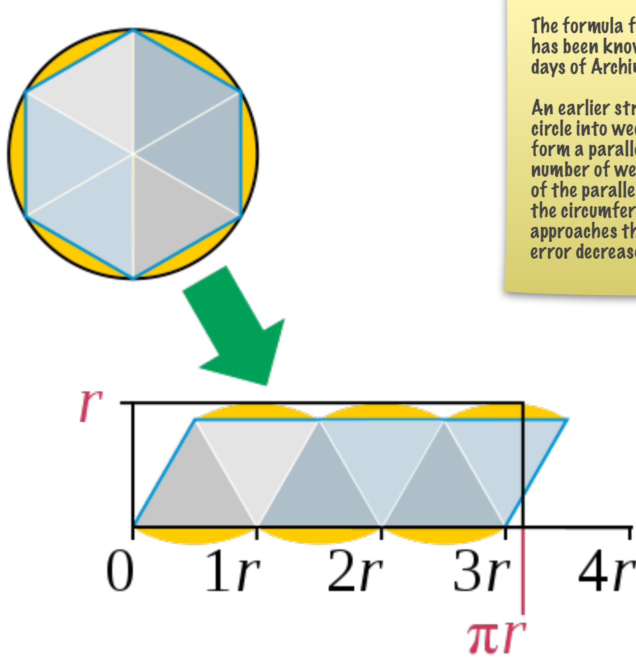
thus the area of a circle with radius r is

$$\begin{aligned}\int_{-r}^{r} 2\sqrt{r^2 - x^2}\,\mathrm{d}x &= r^2 \left(\sin^{-1}\left(\frac{x}{r}\right) + \frac{x}{r}\,\frac{\sqrt{r^2 - x^2}}{r} \right)\Bigg|_{-r}^{r} \\ &= r^2 \left(\sin^{-1}(1) - \sin^{-1}(-1)\right) \\ &= \pi\, r^2\end{aligned}$$

key step here was recognizing that

r^2 - x^2

could be made into a trig identity.



The formula for the area of a circle has been known since at least the days of Archimedes.

An earlier strategy was to divide the circle into wedges and arrange to form a parallelogram. As the number of wedges increases, the base of the parallelogram approaches half the circumference and the height approaches the radius. (And the error decreases.)

http://commons.wikimedia.org/wiki/File:Circle_area_by_reassembly.svg

strategy

$\sqrt{a^2 - x^2}$	$x = a \sin\theta$	$1 - \sin^2 \quad = \cos^2$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

example

evaluate

$$\int \frac{\mathrm{d}x}{x^2\sqrt{25 - x^2}}$$

This is Stewart 7.3.7.

Let x=5 sin (theta)

Answer:

- sqrt(25-x^2) / (25 x) + C

example

evaluate

$$\int \frac{\mathrm{d}t}{\sqrt{t^2 - 6t + 13}}$$

This is Stewart 7.3.24.

Complete the square.

Let t-3 = 2 tan(theta)

Remember: the antiderivative of sec x is ln | sec x + tan x | + C

Answer:

ln | sqrt(t^2-6t+13) + t - 3 | + C

review for midterm

coming soon

- midterm on thursday
- read § 7.4
- start extra credit project 2,
  due 16 november @ 6:00 am
- start homework 5 (due next tuesday)